Reaching the South Pole, discovering the end of Ross's Great Ice Barrier, and making the first landing on King Edward VII. Land is a remarkable triple achievement, and the Norwegian expedition has certainly gained results of first-rate geographical value. Dr. Nansen is to be congratulated on the latest success of his school of polar heroes.

THE TEACHING OF MATHEMATICS.[1]

THE papers before us on "The Teaching of Mathematics in the United Kingdom" are published by the Board of Education as special reports on educational subjects. Each paper of the series (eleven papers are now before us) is written by an expert on the particular subject he treats, and their substantial agreement on educational principles shows the revolution which has taken place in the last decade, and is still taking place in mathematical education.

Last century the subject was taught on the most conventional lines. Few thought of comparing the values, for either mental discipline or knowledge, of different portions of the subject or of different methods of teaching. Such books as the "Inventional Geometry" of Herbert Spencer's father proves the existence of occasional thoughtful men; but in the deadness of the time such books were lost sight of until rediscovered to-day.

The reformers of the later nineteenth century dealt with rigour of proof and completeness of logical development. They aimed at doing for other branches of mathematics what Euclid had done for geometry. A system of mathematics in which the whole subject develops by irrefragable reasoning from a small number of assumptions is a lofty ideal and is an entrancing occupation for certain mature minds; but the school is no place for it. The examination in recent years of attempts at such a system, Euclid's included, leads to the view that no system can do more than approximate roughly to the ideal; the statement of the preliminary assumptions cannot be made complete or the logical development rigorous. This conclusion has added strength to the arm of the band of reformers who hold that this ideal, even if attainable, is out of place in the school.

These reformers recognise that the boy's mind is not the adult mind writ small, that reasoning power develops from an approximate zero in the infant to something far short of perfection in the adult; perfection of reasoning not being attained even in the greatest mathematicians. Consequently they replace this ideal of logical perfection by the ideal of a course suited at every age to the mental development of that age, both in matter and in method of presentation. The matter must in the earliest years be entirely concrete, and must gradually become more abstract with the increasing age and power of the pupil. It should never become entirely abstract, to the exclusion of the concrete, for even in its highest developments mathematics is merely a tool for ultimate application to concrete problems. It is true that it is an economy of labour to have a few mathematicians who work chiefly in the abstract and improve the tool for others to use; but even for these few some knowledge of concrete problems has value for the proper direction of their efforts.

The method of presentation must likewise have regard to the age of the learner. At first there is little reasoning, the teacher's object being to provide in connection with concrete material the abstract ideas for later reasoning, as well as to give precision to such abstract ideas as the pupil already possesses. In the earlier stages evidence is chiefly experimental and intuitional. By appropriate training and increase of years the mind develops and demands more logical evidence. The evidence, suited always to the needs of the pupil, and restricted to the kind which he asks and can grasp, gradually approaches that Euclidean form at which the nineteenth century aimed.

The choice of material out of the various branches of mathematics is important in two ways. The first and obvious criterion is that, other things being equal, the branch which has a direct use in after life, a "bread-and-butter" value, is to be preferred to the branch which has not. The other is that the branch which is the better mental gymnastic is to be chosen. Fortunately these two criteria generally indicate the same branches, the bread-and-butter subject by its relation to life exciting an interest which goes far to give it the preference as mental gymnastic.

The above views run through most of the eleven papers now under review. The battle was first fought in the secondary school, and has been won there as far as the principles are concerned, the questions now at issue being the working out of courses founded on them. The principles are being brought to bear even on the classical boy, naturally enough the last to be affected by a reform in mathematics. In the first paper of the series, "Higher Mathematics for the Classical Sixth Form," Mr. Newbold shows how, in place of the dull committing to memory of Euclid's propositions, such a Form has, by a discussion of problems of everyday life, been given a real and useful grasp of the ideas of the infinitesimal calculus.

In the universities the battle for the new principles is beginning, and Dr. Filon, in his paper on "The Relations of Mathematics and Physics," does yeoman service. As evils requiring regulation he names "(1) mutual misunderstanding due to over-specialisation; (2) the accumulation of uninterpreted material in physics and of abstract concepts in mathematics; (3) the neglect of applied mathematics."

It is unfortunate for the mathematical students at Cambridge that in the rearrangement which admitted physics to a position of consequence, that subject was placed in a tripos distinct from mathematics. Since this estrangement between the two subjects, Cambridge has produced no mathematicians to compare with giants like Kelvin, Stokes, Clerk Maxwell, and Sir J. J. Thomson. Recently a move in the right direction has been made in the attempt to combine the early training of mathematicians, physicists, and engineers; but the success of such a scheme requires more than the revision of regulations.

The third and fourth papers are on "The Teaching of Mathematics in Public Elementary Schools." In these schools the position is somewhat disappointing. The teachers are slow to avail themselves of the free-

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[1] Board of Education. Special Reports on Educational Subjects. "The Teaching of Mathematics in the United Kingdom," being a Series of Papers prepared for the International Commission on the Teaching of Mathematics.

(1) "Higher Mathematics for the Classical Sixth Form." By W. Newbold. Pp. 14. Price 1*d.*

(2) "The Relations of Mathematics and Physics." By Dr. L. N. G. Filon. Pp. ii+9. Price 1*d.*

(3) "The Teaching of Mathematics in London Public Elementary Schools." By P. B. Ballard. Pp. ii+28. Price 2*d.*

(4) "The Teaching of Elementary Mathematics in English Public Elementary Schools." By H. J. Spencer. Pp. 32. Price 2½*d.*

(5) "The Algebra Syllabus in the Secondary School." By C. Godfrey. Pp. 34. Price 2½*d.*

(6) "The Correlation of Elementary Practical Geometry and Geography." By Miss H. Bartram. Pp. ii+8. Price 1*d.*

(7) "The Teaching of Elementary Mechanics." By W. D. Eggar. Pp. ii+13. Price 1*d.*

(8) "Geometry for Engineers." By D. A. Low. Pp. ii+15. Price 1½*d.*

(9) "The Organisation of the Teaching of Mathematics in Public Secondary Schools for Girls." By Miss Louisa Story. Pp. ii+15. Price 1½*d.*

(10) "Examinations from the School Point of View." By Mr. C. Hawkins. Pp. ii+104. Price 9*d.*

(11) "The Teaching of Mathematics to Young Children." By Miss Irene Stephens. Pp. ii+19. Price 1½*d.*

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dom now allowed to them, partly no doubt because of their long discipline under fixed syllabuses, probably partly also because in their work (chiefly arithmetic) there exists no association like the public-spirited Mathematical Association, which has contributed so greatly to the solution of the problem of courses of mathematics in secondary schools.

However, there is progress. Ten years ago, in answer to the simplest question not introduced by one of the mystic words, "multiply," "add," &c., pupils would reply, "I don't know what rule it belongs to." Or they would determine how long Mr. Gladstone lived by multiplying together the years of his birth and death. The two papers now under review show a great advance on that time. And if stocks and shares are still too much in evidence, and the portions of geometry and algebra selected for addition to the curriculum leave something to be desired, there is yet evidence of a great ferment, from which sooner or later good must come. In particular the new Central Schools are full of promise.

The fifth pamphlet of the series, "The Algebra Syllabus in the Secondary School," is a statesmanlike discussion by Mr. Godfrey of the reforms which are at present most urgent in school mathematics. The present ferment in education is acting not only on mathematical masters, but on all other masters, headmasters included. The number of subjects claiming recognition in the school is so great that all cannot be successful in their claims. The inquiry is made with regard to every subject, whether, by reason of its value for knowledge, training, or discipline, it deserves a place in the curriculum or no. Difficult as it is for the mathematician to believe, it is the fact that, so far as concerns non-mathematical boys, the verdict is in danger of going against algebra as at present taught. Many public schools would like to curtail seriously the time given to mathematics.

Something is wrong when headmasters of position and judgment look back on their mathematical training as the "transient but blighting shadow of $x+y$." And those who believe in the value of a mathematical training for all boys must give earnest consideration to the remedy advocated by Mr. Godfrey, a remedy which is already applied in some schools.

Algebra as at present taught is so abstract as to be incomprehensible to the majority of boys. It includes also many portions which lead nowhere in particular, and have no exceptional value as mental discipline. Mr. Godfrey reviews the customary algebra course, and shows severe pruning to be possible and desirable. The time saved by this pruning it is proposed to utilise in giving a useful and educational acquaintance with numerical trigonometry, mechanics on an experimental basis, and the ideas of the infinitesimal calculus. On the calculus Mr. Godfrey's proposals may be usefully studied along with the first pamphlet of this series.

A paper on "The Correlation of Elementary Practical Geometry and Geography" (6) is appropriately included in the series. Geography supplies many illustrations and problems for the use of the mathematical master. In return, when the geography master discusses maps and plans and their making, he finds as a result of the work of his mathematical colleague a readier comprehension on the part of his pupils.

Mr. Eggar's views on "The Teaching of Elementary Mechanics" (7) are shared by the best masters. That they are not more generally put into practice is mainly due to the backwardness of most examining bodies to recognise their merit. It is also partly due to the want of faith of the teacher, for a preliminary or concurrent practical course undoubtedly gives a better grasp, and fits a boy better than the old plan, even for the oldest-fashioned theoretical examination in mechanics. The value of a practical course is placed beyond doubt when the two associations, which represent science and mathematical masters respectively, unite in so strong a recommendation as is contained in the report quoted by Mr. Eggar.

Many words of wisdom are scattered through the paper. One valuable aspiration is that in the future mathematics and physics will be in the hands of one master. For the teaching of mechanics this has the merit of more complete correlation between the practical and theoretical courses. For the mathematical master a knowledge of physics will give a breadth of understanding which is not always found at the present day.

For details of the course Mr. Eggar's paper must be consulted. We will only say here that he wisely follows the historical order in beginning with statics.

(8) "Geometry for Engineers" is less pleasing than the preceding ones. The elaboration of the proposed treatment of conic sections, and (to a less extent) the time it is proposed to devote to synthetic geometry, would appear to necessitate the postponement to a very late stage of subjects so essential to an engineer as mechanics and the infinitesimal calculus. On the other hand, one sympathises with the author's view of the importance of descriptive geometry, both on account of its direct usefulness and on account of the mental training involved in thinking in three dimensions.

(9) "Mathematics in Secondary Schools for Girls." Miss Story's pupils are fortunate in having a mistress so well able to distinguish the gold from the dross. While selection of material is very desirable for boys, it is all-essential for girls. After half a century of attempts to fashion girls' education on the lines fixed by tradition for boys, the country is now realising that it wants to have its girls made into good women and not into inferior men.

(10) "Examinations from the School Point of View" opens with the sound doctrine that qualifying and competitive examinations should be kept distinct, the former being intended to determine which pupils have attained a certain standard, the latter to pick out a certain number of the best. The union of the two tests in a single examination makes the questions too difficult to be a fair test of a moderate general education. On a given range of work fairly complete answers to easy questions are better evidence of ability and knowledge than fragmentary answers to difficult questions.

The author's next proposition is more difficult of acceptance: that in a matriculation examination 70 or 80 per cent. of the candidates should be passed. The object of such an examination being to test fitness to study at a university, the examiners are surely already generous in deciding that 50 per cent. possess that fitness.

Objection to the technical bent of the Army Entrance Examination is possible only in a country which plays at keeping an army. In France and Germany the army is a highly technical profession, and the school education carefully arranged on that understanding. With the author's statement that better ability cannot be secured by stiffening the examination we entirely agree; the remedy lies elsewhere.

In (11) Miss Stephens describes an interesting experiment on the "Teaching of Mathematics to Young Children." The excellent method of the ten-bundle and the hundred-bundle will no doubt lead up to the 100-times table, the 1000-times table, &c., which are more valuable than the 11- and 12-times tables.

DAVID BEVERIDGE MAIR.

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